Bell's inequality
Experimental exercise

Content

Introduction

Purposes of the lab

Besides familiarizing yourselves with the important concept of Bell's inequality in relation to the Einstein-Podolsky-Rosen arguments against quantum mechanics[1], this experimental exercise will also allow you to perform an experiment yourselves using modern (at the time of writing) experimental equipment. Lasers, optics, detectors and coincidence counters are the heart of many experiments concerning quantum optics, quantum cryptography, quantum information theory, quantum computation *&c*.

Tasks & Preparation

The laboration will consist of performing the following tasks:

- Knowing which angles of the half-wave plates that should give the maximum violation of the CHSH-inequality. Which is by the way the maximum theoretical violation we may get with the setup described here? The answers to those questions (angles in degrees) will be the passwords to get into the lab.
- Task 1: Finding out which Bell state is set in the experiment.
- Task 2: Measure the coincidence rates at the half-plate angles which gives maximum violation of the inequality.
- Task 3: Measure the correlation as a function of angle in steps and compare with the theoretical prediction.

If you would like to bring the data home you need to bring an USB memory stick. Most of the data-treatment can be done directly in the laboratory though.

Suggested reading

Wikipedia *caveat lector*: The (english) Wikipedia entry on Bell's inequality and related matters – *taken with a pinch of salt* – is a good complement to the papers suggested in the reading list. As this is, by some, considered to be an unsettled matter the content may be colored by other than purely scientific considerations.

(i) D. Dehlinger, M. W. Mitchell, *Am. J. Phys.*, **70**, 903 (2002)

(ii) D. Dehlinger, M. W. Mitchel, *Am. J. Phys.* **70**,, 898 (2002)

(iii) M. A. Rowe, *et al.*, Nature, **409**, 791 (2001)

[1] A. Einstein, B. Podolsky, N. Rosen, *Phys. Rev. Lett* **47**, 777 (1935)

In the lab

Safety considerations

A few simple rules will carry us a long way here:

DO NOT LOOK INTO THE LASER BEAM!

THINK before you do anything:

Do not touch *anything* but the two half-wave plates after the yellow laser fibre.

Switch off the detectors before turning on the light, switch on the detectors when it is dark.

Always turn the half-wave plates **counterclockwise**

The assistants will show you how to properly turn the half-wave plates when it is dark and how to switch the detectors on and off.

Experimental set-up

The photon source will be set-up by the assistants beforehand.

You do not need to – ***and you will not*** – touch

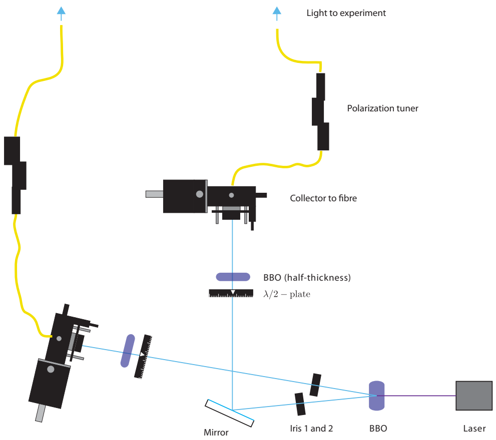
or "adjust" anything on it.

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|H_1V_2\rangle \pm |V_1H_2\rangle\right)$$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|H_1H_2\rangle \pm |V_1V_2\rangle\right)$$

The BBO crystal produce a definite Bell state by down-converting one UV photon:

$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}\left(|H_1V_2\rangle + |V_1H_2\rangle\right)$$



The BBO/2 crystals before the fibre allows for tuning the phase – both to remove any resultant phase owing to different path lengths in the BBO crystal (which could make a path-selection measurement possible) and to select from the palette of different possible Bell states (written above the photon source). One of the states will be pre-selected for the exercise.

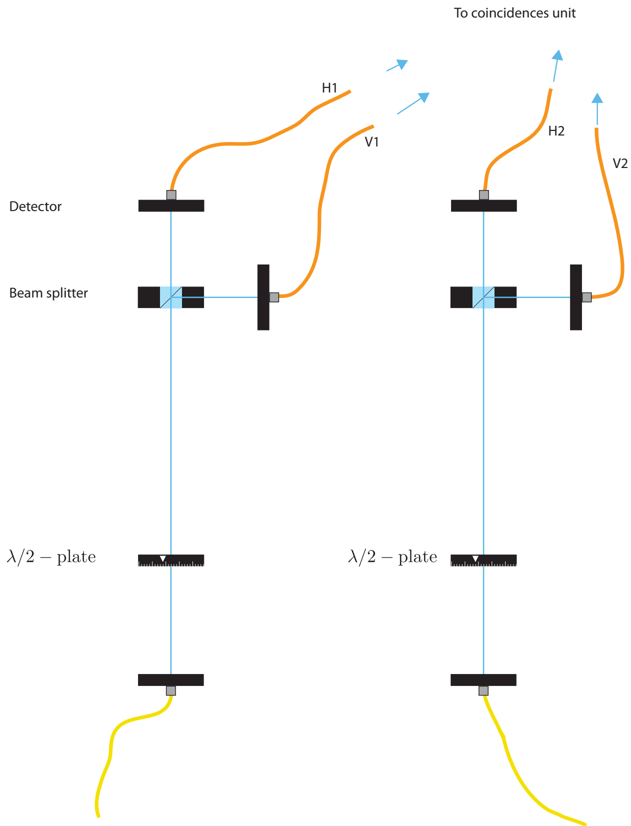
The twist of the fibres can be adjusted to align the polarizations of the fibres (so that no extra phase is introduced by the fibre). Obviously any tampering with that "tuning" will have severe impact on the results of the experiment.

The detector part is described below.

What can we measure?

You are allowed to rotate the half-wavelength plates, switch the laser on and off, switch the detector on and off and the light on and off (see above).

On the computer you can read the number of photon-counts and the number of coincidence counts between the different arms, *e.g.* how many photons of H polarization are counted with an accompanying V polarization photon (*i.e.* N(HV), N(VH), N(HH), N(VV)).



The beamsplitters are polarizing and let horizontally polarized light through and vertically polarized light are reflected.

The first task is to determine which Bell state that the photon source produce – this can be inferred from the behavior of the coincidence countrate. Be careful to note what the "zero" setting is on the half-wave plates before turning them. **Turn them counterclockwise.**

The second task is to do a Bell test. This require measurements of the coincidence rates at four different polarization settings.

The table below needs to be filled in.

Afterwards the results can be analyzed in an Octave (Matlab) script on the measurement computer. From this you will get both a value for S and the standard deviation.

Angle 1	*Angle 2*	*HH*	*HV*	*VH*	*VV*

theoretical max(S_{HVT})=

theoretical max(S_{QM})=

Measured S_{QM}=_______________ Standard deviation: ________________

If any, by how many deviations did the experiment violate the hidden variable theories' prediction of |S|≤2?

The last task is to measure the value of the correlation E for a range of angles, to compare with the quantum mechanical expression for the correlation.

$$E(\theta_a, \theta_b) = -\cos[2(\theta_b - \theta_a)]$$

The report

Outline

Each group need to hand in one report. Each student should have hers/his own frontpage with title, coursecode, date and name on it. This coverpage will serve as your proof, when it is signed by the assistants, that you completed the lab-work for the course. The assistants will report it also of course, but this is your confirmation that it was reported.

The report should describe and contain:

- the intention of the experiment(s) together with some background.
- How the experiment(s) were performed, *i.e.* describe the main components of the set-up and what their purposes are.
- The results and data obtained from your measurements: which Bell state did you perform the subsequent experiments on? How did you find that out? The data from the Bell test task (task 2) should be given in a table, the last measurement can be presented in a graph together with the theoretical curve.
- The derivation of the correlation as function of the angle settings (see next page).
- A discussion and conclusion.
- Key references.

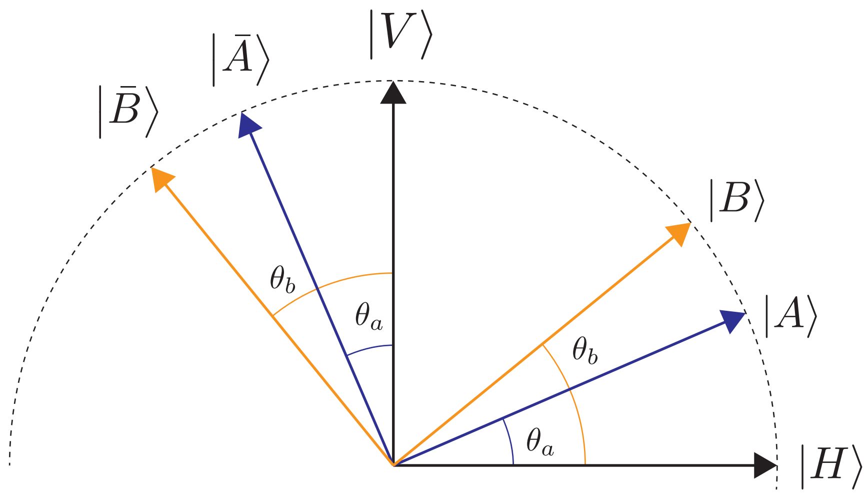
Remember that it is not a scientific paper you are writing, you only need to tell us what you did and walk us through what you did and the results you obtained in about 4-5 A4 pages. Tip: consider writing for a student or a teacher at another university that have got the set-up in the same condition as you got it (i.e. with a Bell state prepared but unknown to you) and needs to perform the experiment.

Schematic pictures of the set-up is included below.

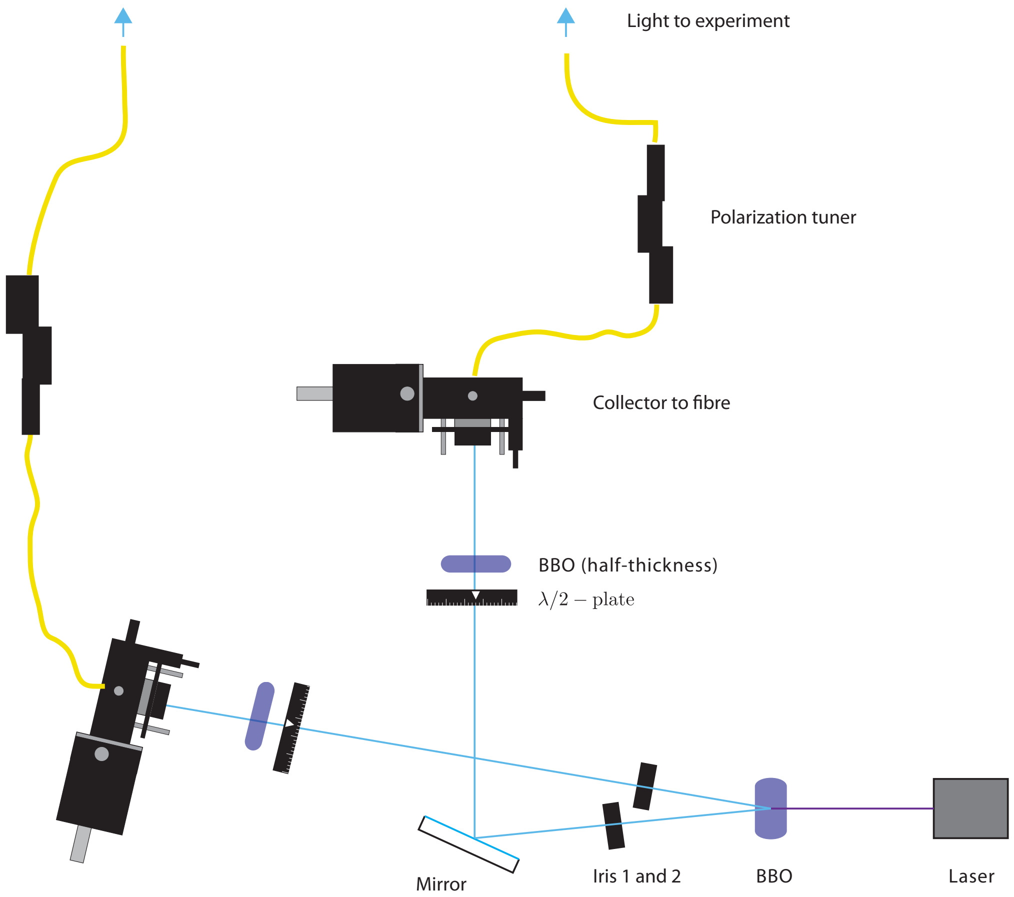
Theoretical consideration: show that the quantum mechanical fomulation of the correlation between the measurements at the two analyzers can be expressed as, (consider how the Bell state used will manifest itself in the two different bases):

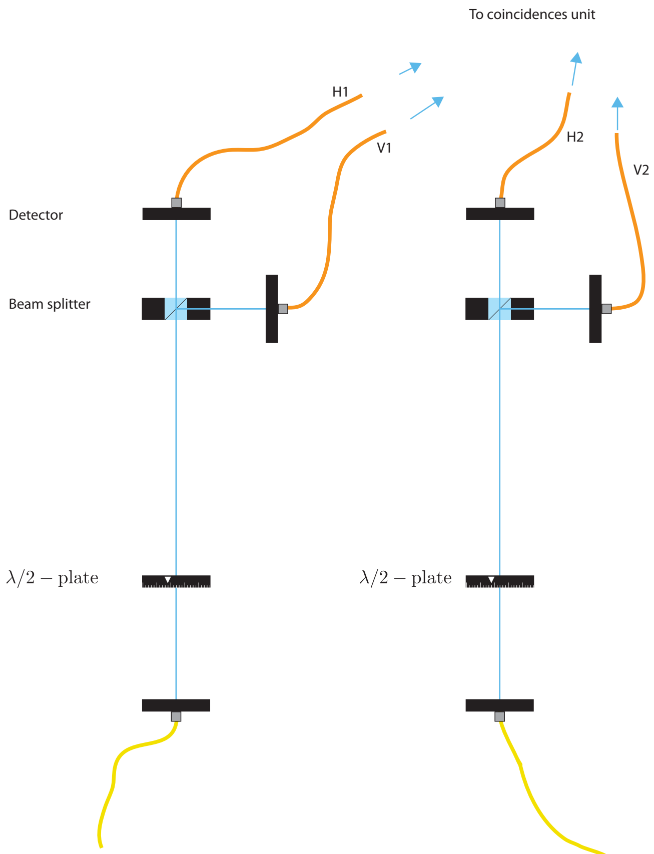
$$E(\theta_a, \theta_b) = -\cos[2(\theta_b - \theta_a)]$$

$$E = P(B, A) + P(\bar{B}, \bar{A}) - P(B, \bar{A}) - P(\bar{B}, A)$$



$\{|H\rangle, |V\rangle\}$ $\{|A\rangle, |\bar{A}\rangle\}$ $\{|B\rangle, |\bar{B}\rangle\}$


Light to experiment
Polarization tuner
Collector to fibre
BBO (half-thickness)
$\lambda/2$ – plate
Mirror
Iris 1 and 2
BBO
Laser


To coincidences unit
H1
V1
H2
V2
Detector
Beam splitter
$\lambda/2$ – plate
$\lambda/2$ – plate